King Fahd University of Petroleum & Minerals
Department of Information and Computer Science

Solution

Question	1	2	3	4	5	6	7	8	Total
Max	5	10	5	5	10	10	10	45	100
Earned									

Question 1: [5 Points] [CLO 1] Propositional Logic
State the contrapositive of the conditional statement: "If n^2 is even, then n is even."

If n is odd then n^2 is odd.

Question 2: [10 Points] [CLO 2] Introduction to Proofs
Given that n is integer. Prove that if $n^3 + 5$ is odd then n is even.

Prove by contraposition: we need to prove If n is odd then $n^3 + 5$ is even.
$n = 2k +1$. So $n^3 + 5 = (2k+1)^3 + 5 = 8k^3 + 12k^2 +6k + 6 = 2(4k^3 + 6k^2 +3k + 3) = 2r$.
This shows that $n^3 + 5$ is even. This completes the proof of the contraposition.
So this proves that If n3 + 5 is odd then n is even.

Question 3: [5 points] [CLO 1] Predicates and Quantifiers
Let p(x) and q(x, y) be the predicates p(x): x is in our class, q(x, y): x knows y,
where the domain of x and y is the set of all students in KFUPM. Translate the following to English:
$\exists x \forall y [(p(y) \rightarrow \neg q(x, y)]$
There exist a student who does not know any student in our class.

Question 4: [5 points] [CLO 1] Predicates and Quantifiers
Let h(x, y) be the statement x can help y, where the domain consists of all people in the world. Use quantifier to express the statement: “Every one can be helped by somebody”.
$\forall y \exists x\, h(x, y)$

Question 5: [10 points] [CLO 2] Introduction to Proofs
Use the logical equivalence laws to show that $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ is a tautology. Clearly show all steps. (Do not use truth tables)

Proof: $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$

$\equiv \neg[(p \rightarrow q) \wedge \neg q] \vee \neg p$	Implication Definition
$\equiv \neg[(\neg p \vee q) \wedge \neg q] \vee \neg p$	Implication Definition
$\equiv (\neg (\neg p \vee q) \vee \neg\neg q) \vee \neg p$	DeMorgan’s
$\equiv ((\neg\neg p \wedge \neg q) \vee \neg\neg q) \vee \neg p$	DeMorgan’s
$\equiv ((p \wedge \neg q) \vee q) \vee \neg p$	Double Negation (twice)
$\equiv \neg p \vee (q \vee (p \wedge \neg q))$	Commutative (three times)
$\equiv \neg p \vee ((q \vee p) \wedge (q \vee \neg q))$	Distributive
$\equiv \neg p \vee ((q \vee p) \wedge T)$	Negation
$\equiv \neg p \vee (q \vee p)$	Domination
$\equiv \neg p \vee (p \vee q)$	Commutative
$\equiv q \vee (\neg p \vee p)$	Associative/ Commutative
$\equiv q \vee T$	Negation
$\equiv \mathbf{T}$	Domination law

$\therefore [(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ is a tautology

Question 6: [10 points] [CLO 2] Rules of Inference and Proof
Using the inference rules and equivalences show that the premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\neg s$ imply the conclusion that $q \rightarrow r$.

Given

[01] $q \rightarrow (u \wedge t)$ Premise
[02] $(p \wedge t) \rightarrow (r \vee s)$ Premise
[03] $\neg s$ Premise
[04] $u \rightarrow p$ Premise

Want to show

$q \rightarrow r$

Proof

[05] $q \rightarrow u$ Simplification on [01] as $q \rightarrow (u \wedge t)$ is equivalent to $(q \rightarrow u) \wedge (q \rightarrow t)$
[06] $q \rightarrow t$ Simplification on [01] as $q \rightarrow (u \wedge t)$ is equivalent to $(q \rightarrow u) \wedge (q \rightarrow t)$
[07] $q \rightarrow p$ Hypothetical Syllogism on [05] and [04]
[08] $q \rightarrow (p \wedge t)$ Conjunction on [07] and [06]
[09] $(p \wedge t) \rightarrow r$ Disjunctive Syllogism on [02] and [03]
[10] $q \rightarrow r$ Proof Hypothetical Syllogism on [08] and [09]

A second approach

[05] q Assumption
[06] $u \wedge t$ Modus Ponens on [05] and [01]
[07] u Simplification on [06]
[08] t Simplification on [06]
[09] p Modus Ponens on [07] and [04]
[10] $(p \wedge t)$ Conjunction on [09] and [08]
[11] $(r \vee s)$ Modus Ponens on [10] and [02]
[12] r Disjunctive Syllogism on [11] and [03]
[13] $q \rightarrow r$ Proof [05] and [12]

Question 7: [10 points] [CLO 2] Set identities
Using set identities prove: $A \cup B = (A \cap \overline{B}) \cup (\overline{A} \cap B) \cup (A \cap B)$

$(A \cap \overline{B}) \cup (\overline{A} \cap B) \cup (A \cap B)$ Premise
$= (A \cap \overline{B}) \cup [(A \cup \overline{A}) \cap B]$ Distributive
$= (A \cap \overline{B}) \cup (\mathbf{U} \cap B)$ Complement
$= (A \cap \overline{B}) \cup B$ Identity
$= (A \cup B) \cap (\overline{B} \cup B)$ Distributive
$= (A \cup B) \cap \mathbf{U}$ Complement
$= (A \cup B) \cap \mathbf{U}$ Identity

Question 8: [45 Points] [CLO #1] Indicate whether the given sentence is true or false. In the answer column write either ✓ for "true" or × for "false".

Statement	Answer
1. Not every set is a subset of itself.	×
2. if A and B are two sets with different power sets then $A \neq B$.	✓
3. If A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h} then A - B = { }.	✓
4. $A - B = \{x \mid x \in A \land x \notin B\}$.	✓
5. Two sets are called disjoint if their intersection is the empty set.	✓
6. $\overline{A \cap B} = \{x \mid \neg(x \in A \land x \in B)\}$	✓
7. If The bit strings for the sets {1, 2, 3, 4, 5} and {1, 3, 5, 7, 9} are 1111100000 and 1010101010, respectively, then the intersection of these sets is 1010100000.	✓
8. A function f is decreasing if $\forall x \forall y (x < y \rightarrow f(x) \geq f(y))$.	✓
9. If f Is the function from $\mathbb{Z}$ to $\mathbb{Z}$ with $f(x) = x + 1$ then f is invertible.	✓
10. $\lfloor x \rfloor = n$ if and only if $x \leq n < x + 1$.	×
11. $f(n) = 1/(n^2 - 4)$ is not a function from $\mathbb{Z}$ to $\mathbb{R}$	✓
12. $f(n) = \lceil n/2 \rceil$ from $\mathbb{Z}$ to $\mathbb{Z}$ is not one-to-one function.	✓
13. The function $f(x) = -3x^2 + 7$ from $\mathbb{R}$ to $\mathbb{R}$ is neither one-to-one nor onto, therefore it is not a bijection.	✓
14. If $f(x) = x^2 + 1$ and $g(x) = x + 2$ are functions from $\mathbb{R}$ to $\mathbb{R}$ then (f o g) is $x^2 + 4x + 5$.	✓
15. One way to express the statement "Every user has access to an electronic mailbox." using predicates, quantifiers, and logical connectives is $\forall x(\text{user}(x) \rightarrow \exists y(\text{electronic\_mailbox}(y) \land \text{has\_access}(x, y)))$	✓
16. There are 64 rows appear in the truth table for the compound propositions $(p \rightarrow r) \lor (\neg s \rightarrow \neg t) \lor (\neg u \rightarrow v)$.	✓
17. $(p \lor q \lor r) \land (\neg p \lor \neg q \lor \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.	✓
18. $\neg(p \rightarrow q) \rightarrow p$ is a tautology.	✓
19. $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are not logically equivalent.	×
20. Given the domain is the real numbers, the statement $\forall y \neq 0\ (y^3 \neq 0)$ means "The cube of every nonzero real number is nonzero."	✓
21. If the domain for all variables consists of all integers then $\forall x \exists y (y^2 = x)$.	×

EQUIVALENCES AND IMPLICATION EQUIVALENCES

Double negation law: $\neg(\neg p) \equiv p$

Identity laws: $p \vee \mathbf{F} \equiv p$, $p \wedge \mathbf{T} \equiv p$

Domination laws: $p \vee \mathbf{T} \equiv \mathbf{T}$, $p \wedge \mathbf{F} \equiv \mathbf{F}$

Negation laws: $p \vee \neg p \equiv \mathbf{T}$, $p \wedge \neg p \equiv \mathbf{F}$

Idempotent laws: $p \vee p \equiv p$, $p \wedge p \equiv p$

Commutative laws: $p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$

Associative laws: $p \vee (q \vee r) \equiv (p \vee q) \vee r$,
$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

Distributive laws: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$,
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Absorption laws: $p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$

DeMorgan's laws: $\neg(p \vee q) \equiv \neg p \wedge \neg q$,
$\neg(p \wedge q) \equiv \neg p \vee \neg q$

1. $p \to q \equiv \neg p \vee q$
2. $p \to q \equiv \neg q \to \neg p$
3. $p \vee q \equiv \neg p \to q$
4. $p \wedge q \equiv \neg(p \to \neg q)$
5. $\neg(p \to q) \equiv p \wedge \neg q$
6. $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
7. $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
8. $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
9. $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$
10. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
11. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
12. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
13. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

RULES OF INFERENCE

p

$\therefore\ p \vee q$ (*Addition*)

$p \wedge q$

$\therefore\ p$ (*Simplification*)

p
q

$\therefore\ p \wedge q$ (*Conjunction*)

p
$p \to q$

$\therefore\ q$ (*Modus ponens*)

$\neg q$
$p \to q$

$\therefore\ \neg p$ (*Modus tollens*)

$p \to q$
$q \to r$

$\therefore\ p \to r$ (*Hypothetical syllogism*)

$p \vee q$
$\neg p$

$\therefore\ q$ (*Disjunctive syllogism*)

$p \vee q$
$\neg p \vee r$

$\therefore\ q \vee r$ (*Resolution*)

Odus ponens

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws